COMPLEX ANALYSIS, HW # 5

Section 16, problems 1, 2, 3, and these problems:

Problem 1.

Find the integral
$$\int_0^\infty \frac{\cos(2x)}{1+x^2} dx.$$

Problem 2.

Consider a series $f(z) = \sum_{n=0}^\infty \frac{z^n}{n+1}$.

a) What is the radius of convergence of this series?

b) Take the power series for f near the point $\frac{i}{2}$, i.e. $f(z) = \sum_{n=0}^\infty a_k(z - \frac{i}{2})^k$. What is the radius of convergence of this series?

Problem 3.

Let $f(z) = \sum_{k=0}^\infty a_k z^k$ be an entire function. Find the integral

$$\int_{|z|=1} f\left(\sin\left(\frac{1}{z}\right)\right) dz$$

Problem 4.

Let $f_j : D(0,1) \to D(0,1)$ be holomorphic for each $j \in \mathbb{N}$. Suppose

$$\lim_{z \to \infty} f_j(0) = 1.$$

Show that $f_j(z) \to 1$ uniformly on compact subsets of $D(0,1)$.