Complex Analysis, HW # 1

Section 10, problems 9, 10, 12, **and these problems:**

Problem 1.

Find all roots of the equation $2z + \sin z = 0$ in the unit disk.

Problem 2.

Compute the area of the image of the unit disc $D = \{z \mid |z| < 1\}$ under the map $f(z) = z + \frac{z^2}{2}$.

Problem 3.

Suppose a function $f : \overline{D} \to \mathbb{C}$ is continuous and holomorphic in D. Suppose also that for any $z \in \partial D$ we have $\mathbf{Re} f(z) = (\mathbf{Im} f(z))^2$. Prove that f is a constant.

Problem 4.

Let $U \subset \mathbb{C}$ be a connected (but not necessarily simply connected) open set, and γ be a closed curve in U. Suppose that for any function f holomorphic on U we have

$$\oint_\gamma f(z)dz = 0.$$

Does it imply that γ is homotopic to a constant curve?