

COMPLEX ANALYSIS MATH 220A

Final Exam (sample)

Problem 1.

Does there exist a conformal automorphism φ of the unit disc such that $\varphi(1/2) = 0$ and $\varphi(0) = \frac{i}{3}$?

Problem 2.

Let f and g be analytic on a bounded domain D and continuous on its closure. Show that $|f(z)| + |g(z)|$ attains its maximum on the boundary of D .

Problem 3.

Let $D = \{z : |z| < 1\}$, and let $f : D \rightarrow D$ have a zero of order n at zero. Show that $|f(z)| \leq |z|^n$ on D and $|f^{(n)}| \leq n!$.

Problem 4.

Find the number of solutions of the equation $2z^8 + 15 + 16z = 0$ in the left-half plane $D = \{z : \operatorname{Re} z < 0\}$.

Problem 5.

Evaluate the integral

$$\int_0^\infty \frac{1}{(1+x^2)x^{1/2}} dx$$