MATH 3D, DIFFERENTIAL EQUATIONS
SAMPLE FINAL

Problem 1.

Find the general solution of the equation

$$y'' - 4y' + 3y = \frac{e^x}{e^x + 1}$$

Problem 2.

Find e^{At}, where

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

Problem 3.

Determine the inverse Laplace transform $\mathcal{L}^{-1}(F(s))$, where

$$F(s) = \frac{e^{-7s}}{s^2 + 9}$$

Problem 4.

Solve the initial value problem

$$\begin{cases} y'' + y = f(x), \\ y(0) = 1, \\ y'(0) = 0, \end{cases}$$

where $f(x) = \begin{cases} 1, & \text{if } x < 1; \\ 2, & \text{if } x \geq 1. \end{cases}$

Problem 5.

Find the solution of the system of differential equations

$$\begin{cases} \dot{x} = x + y, \\ \dot{y} = 9x + y, \end{cases}$$

that satisfies the initial conditions $x(1) = 3, y(1) = 3$.

Laplace Transforms

$f(x)$	$F(s) = \mathcal{L}(f(x)) = \int_0^\infty e^{-sx} f(x)dx$
1	$\frac{1}{s},\ s > 0$
x	$\frac{1}{s^2},\ s > 0$
$x^n, n = 1, 2, \ldots$	$\frac{n!}{s^{n+1}}, s > 0$
e^{ax}	$\frac{1}{s-a}, s > a$
$\sin(ax)$	$\frac{a}{s^2+a^2}, s > 0$
$\cos(ax)$	$\frac{s}{s^2+a^2}, s > 0$
$e^{ax}g(x)$	$G(s-a)$, where $G(s) = \mathcal{L}(g(x))$
$x^n g(x)$	$(-1)^n \frac{d^n}{ds^n} G(s)$, where $G(s) = \mathcal{L}(g(x))$
$u(x-c)g(x-c)$	$e^{-cs}G(s)$, where $G(s) = \mathcal{L}(g(x))$

Also,

$$\mathcal{L}(y'(x)) = sY(s) - y(0),$$
$$\mathcal{L}(y''(x)) = s^2Y(s) - sy(0) - y'(0),$$

where $Y(s) = \mathcal{L}(y(x))$.