# INTRO DIFFERENTIAL EQUATIONS

## Practice Midterm Exam with answers

## Version 1.

Problem 1.
Find a general solution of the equation

$$(\sin x + y)\frac{dy}{dx} + y\cos x - x^2 = 0$$

Answer: $y\sin x - \frac{x^3}{3} + \frac{y^2}{2} = C$

Problem 2.
Find the solution of the initial value problem

$$x\frac{dy}{dx} + x^2 + xy - y = 0, \;\; y(1) = 0.$$

Answer: $y(x) = x(e^{1-x} - 1)$

Problem 3.
Find a general solution of the equation

$$y'' - 2y' + y = \frac{e^x}{x}$$

Answer: $y(x) = e^x(x\ln|x| + C_1 x + C_2)$

Problem 4.
What second order linear homogeneous differential equation with constant coefficients has a solution $y(x) = e^{2x}\sin 4x$?

Answer: $y'' - 4y' + 20y = 0$

Problem 5.
Find a general solution of the equation

$$y'' + 4y = e^x \sin 2x$$

Answer: $y(x) = C_1\cos(2x) + C_2\sin(2x) + \frac{1}{17}e^x\sin(2x) - \frac{4}{17}e^x\cos(2x)$

## Version 2.

Problem 1.
Find a general solution of the equation

$$y \sin x + y' \cos x = 1$$

Answer: $y(x) = C \cos x + \sin x$

Problem 2.
Find the solution of the initial value problem

$$y - y' = y^2 + xy', \;\; y(1) = 0.$$

Answer: $y(x) \equiv 0$

Problem 3.
Find a general solution of the equation

$$y'' + 3y' + 2y = \frac{1}{e^x + 1}$$

Answer: $y(x) = (e^{-x} + e^{-2x}) \ln(e^x + 1) + C_1 e^{-x} + C_2 e^{-2x}$

Problem 4.
What second order linear homogeneous differential equation with constant coefficients has a solution $y(x) = 2e^x \sin x$?

Answer: $y'' - 2y' + 2y = 0$

Problem 5.
Find a general solution of the equation

$$y''' - y'' - y' + y = 0$$

Answer: $y(x) = C_1 e^{-x} + C_2 e^x + C_3 x e^x$

## Version 3.

Problem 1.
Find a general solution of the equation

$$xy' = y - xe^{\frac{y}{x}}$$

Answer: $y(x) = -x \ln \ln(Cx)$

Problem 2.
Find the solution of the initial value problem

$$y = x(y' - x\cos x), \;\; y(\pi) = \pi.$$

Answer: $y(x) = x(1 + \sin x)$

Problem 3.
Find a general solution of the equation

$$y'' + y = \frac{1}{\sin x}$$

Answer: $y(x) = (C_1 + \ln|\sin x|)\sin x + (C_2 - x)\cos x$

Problem 4.
What second order linear homogeneous differential equation with constant coefficients has a solution $y(x) = 5e^{-x}\cos x$?

Answer: $y'' + 2y' + 2y = 0$

Problem 5.
Find a general solution of the equation

$$y'' - 5y' + 4y = 4x^2e^{2x}$$

Answer: $y(x) = C_1e^x + C_2e^{4x} - (2x^2 - 2x + 3)e^{2x}$

## Version 4.

Problem 1.
Find a general solution of the equation

$$x - y - 1 + (y - x + 2)y' = 0$$

Answer: $(y - x + 2)^2 + 2x = C$

Problem 2.
Find a general solution of the equation

$$xy' + 2y + x^5y^3e^x = 0$$

Answer: $y^{-2} = x^4(2e^x + C)$

Problem 3.
Find a general solution of the equation

$$(2x + y)dy = ydx + 4\ln y\, dy$$

Answer: $x = 2\ln\, y - y + 1 + Cy^2$

Problem 4.
Find a general solution of the equation

$$\frac{d^5y}{dx^5} + 8\frac{d^3y}{dx^3} + 16\frac{dy}{dx} = 0$$

Answer: $y(x) = C_1 + (C_2 + C_3x)\cos 2x + (C_4 + C_5x)\sin 2x$

Problem 5.
Find the solution of the initial value problem

$$y''' - 3y' - 2y = 9e^{2x}; \quad y(0) = 0,\ y'(0) = -3,\ y''(0) = 3$$

Answer: $y(x) = (x - 1)(e^{2x} - e^{-x})$