INTRO DIFFERENTIAL EQUATIONS

Midterm Exam

Friday, May 6, 2011 — 1:00 pm - 1:50 pm

Problem 1.

Find a general solution of the equation

$$e^{-y}dx - (2y + xe^{-y})dy = 0$$

Answer: $x = Ce^y + y^2e^y$

Problem 2.

Solve the initial value problem

$$\begin{cases} (y^2 - 2xy)dx + x^2dy = 0, \\ y(1) = 1. \end{cases}$$

Answer: $y(x) = x$

Problem 3.

Find a general solution of the equation

$$2x(x^2 + y)dx = dy$$

Answer: $y = Ce^{x^2} - x^2 - 1$

Problem 4.

Find a general solution of the equation

$$y'' + y = x \sin x$$

Answer: $y(x) = C_1 \cos x + C_2 \sin x + \left(-\frac{x^2}{4} \cos x + \frac{x}{4} \sin x\right)$

Problem 5.

Solve the initial value problem

$$\begin{cases} \frac{d^4y}{dx^4} - y = 0, \\ y(0) = 1,\ y'(0) = 1,\ y''(0) = -1,\ y'''(0) = -1. \end{cases}$$

Answer: $y = \sin x + \cos x$