MULTIVARIABLE CALCULUS   MATH 2D

Midterm Exam I with answers

Problem 1.

Determine whether the given vectors are orthogonal, parallel, or neither.

a) $< 4, 1, 2 >$ and $< -1, 0, 2 >$

b) $< 3, 2\sqrt{3}, 2 >$ and $< -1, -\frac{2}{\sqrt{3}}, -\frac{2}{3} >$

c) $< 3, 4, 3 >$ and $< -1, 5, 3 >$

Answers:

a) orthogonal;

b) parallel;

c) neither.

Problem 2.

Calculate the given quantity if

$$\bar{v} =< 3, 2, -1 >, \bar{u} =< 1, 0, 2 >, \bar{w} =< -2, 1, 1 >$$

a) $2\bar{v} + 2\bar{u} - \bar{w}$

b) $|\bar{v} \times \bar{u}|$

c) $\bar{v} \cdot (\bar{u} \times \bar{w})$

Answers:

a) $< 10, 3, 1 >$

b) $\sqrt{69}$

c) -17

Problem 3.

Find the equation of the plane through $(1, 2, 3)$ that contains the line

$$x = 2t + 2, \ y = 2t - 3, \ z = -t + 3$$

Answer: $5x + y + 12z = 43$

Problem 4.

Find the distance from the origin to the line

$$x = 3t + 2, \ y = -t - 3, \ z = -t - 1$$

Answer: $\sqrt{\frac{54}{11}}$

Problem 5.

Reduce the equation to one of the standard forms, classify the surface:

$$x^2 - 2y^2 + 3z^2 - 2x - 4y - 6z - 15 = 0$$

Answer: $\frac{(x-1)^2}{a^2} - \frac{(y+1)^2}{b^2} + \frac{(z-1)^2}{c^2} = 1$, where $a^2 = 17, b^2 = 17/2, c^2 = 17/3$. It is a hyperboloid of one sheet.