Midterm Exam I with answers

Problem 1.

Determine whether the given vectors are orthogonal, parallel, or neither.

a) < 3, 2, 1 > and < -1, 0, 3 >

b) < 3,
$$\sqrt{3}$$
, 5 > and < -1, $-\frac{1}{\sqrt{3}}$, $-\frac{5}{3}$ >

c) < 3, 4, 3 > and < -3, -2, 1 >

Answers:

a) orthogonal;

- b) parallel;
- c) neither.

Problem 2.

Calculate the given quantity if

 $\bar{v} = <3, 2, 1>, \bar{u} = <-2, 0, -1>, \bar{w} = <2, 1, 1>$

- a) $2\bar{v} + 2\bar{u} \bar{w}$
- b) $|\bar{v} \times \bar{u}|$

c) $\bar{v} \cdot (\bar{u} \times \bar{w})$

Answers:

a) < 0, 3, −1 >

b) $\sqrt{21}$

c) 1

Problem 3.

Find the equation of the plane through (1, 2, 3) that contains the line

x = 3t + 2, y = t - 3, z = -t - 1

Answer: 9x - 11y + 16z = 35

Problem 4.

Find the distance from the origin to the line

$$x = 3t + 2, y = t - 1, z = t + 2$$

Answer: $\sqrt{\frac{50}{11}}$

Problem 5.

Reduce the equation to one of the standard forms, classify the surface:

$$x^2 - 2y^2 - 2x - 4y + z - 15 = 0$$

Answer: $\frac{(x-1)^2}{a^2} - \frac{(y+1)^2}{b^2} = -(z-14)$, where $a^2 = 1, b^2 = 1/2$. It is a hyperbolic paraboloid.