Midterm Exam I with answers

Problem 1.

Determine whether the given vectors are orthogonal, parallel, or neither.

- a) < 2, 1, 3 >and < 0, -3, 1 >
- b) $< 3, \sqrt{3}, 2 >$ and $< -1, -\frac{1}{\sqrt{3}}, -\frac{2}{3} >$
- c) < 2, 5, 3 >and < -1, 5, 1 >

Answers:

- a) orthogonal;
- b) parallel;
- c) neither.

Problem 2.

Calculate the given quantity if

$$\bar{v} = <2, 1, 3>, \bar{u} = <2, 1, 0>, \bar{w} = <-2, 1, -1>$$

- a) $2\bar{v} + 2\bar{u} \bar{w}$
- b) $|\bar{v} \times \bar{u}|$
- c) $\bar{v} \cdot (\bar{u} \times \bar{w})$

Answers:

- a) < 10, 3, 7 >
- b) $\sqrt{45}$
- c) 12

Problem 3.

Find the equation of the plane through (2,1,3) that contains the line

$$x=t+2,\;y=2t-3,\;z=-t+1$$

Answer: 4x - y + 2z = 13

Problem 4.

Find the distance from the origin to the line

$$x = 2t + 1, y = t - 1, z = t - 5$$

Answer: $\sqrt{\frac{73}{3}}$

Problem 5.

Reduce the equation to one of the standard forms, classify the surface:

$$x^2 + 2y^2 - 2x - 4y - z - 15 = 0$$

Answer: $\frac{(x-1)^2}{a^2} + \frac{(y+1)^2}{b^2} = (z+14)$, where $a^2 = 1, b^2 = 1/2$. It is an elliptic paraboloid.