Midterm Exam I (with answers)

Problem 1.

Determine whether the given vectors are orthogonal, parallel, or neither.

a)
$$< 1, 2, 3 >$$
 and $< -3, 0, 1 >$

b)
$$< 3, \sqrt{3}, 1 >$$
 and $< -1, -\frac{1}{\sqrt{3}}, -\frac{1}{3} >$

c)
$$< 2, 4, 3 >$$
and $< -3, 5, 1 >$

Answers:

- a) orthogonal;
- b) parallel;
- c) neither.

Problem 2.

Calculate the given quantity if

$$\bar{v} = <1, 2, 3>, \bar{u} = <2, 0, 1>, \bar{w} = <-2, -1, 1>$$

- a) $2\bar{v} + 2\bar{u} \bar{w}$
- b) $|\bar{v} \times \bar{u}|$
- c) $\bar{v} \cdot (\bar{u} \times \bar{w})$

Answers:

- a) < 8, 5, 7 >
- b) $\sqrt{45}$
- c) -13

Problem 3.

Find the equation of the plane through (1,2,3) that contains the line

$$x = t + 2, \ y = 2t - 3, \ z = -t + 1$$

Answer: 9x - y + 7z = 28

Problem 4.

Find the distance from the origin to the line

$$x = 2t + 2, y = t - 3, z = t - 4$$

Answer: $\sqrt{\frac{55}{2}}$

Problem 5.

Reduce the equation to one of the standard forms, classify the surface:

$$x^2 + 2y^2 + 3z^2 - 2x - 4y - 6z - 15 = 0$$

Answer: $\frac{(x-1)^2}{a^2} + \frac{(y-1)^2}{b^2} + \frac{(z-1)^2}{c^2} = 1$, where $a^2 = 21, b^2 = 21/2, c^2 = 7$. It is an ellipsoid.