## Midterm Exam I (with answers)

## Problem 1.

Determine whether the given vectors are orthogonal, parallel, or neither.
a) $\langle 1,2,3\rangle$ and $\langle-3,0,1\rangle$
b) $\langle 3, \sqrt{3}, 1\rangle$ and $\left.<-1,-\frac{1}{\sqrt{3}},-\frac{1}{3}\right\rangle$
c) $\langle 2,4,3\rangle$ and $\langle-3,5,1\rangle$

Answers:
a) orthogonal;
b) parallel;
c) neither.

## Problem 2.

Calculate the given quantity if

$$
\bar{v}=<1,2,3>, \bar{u}=<2,0,1>, \bar{w}=<-2,-1,1>
$$

a) $2 \bar{v}+2 \bar{u}-\bar{w}$
b) $|\bar{v} \times \bar{u}|$
c) $\bar{v} \cdot(\bar{u} \times \bar{w})$

Answers:
a) $\langle 8,5,7\rangle$
b) $\sqrt{45}$
c) -13

## Problem 3.

Find the equation of the plane through $(1,2,3)$ that contains the line

$$
x=t+2, y=2 t-3, z=-t+1
$$

Answer: $9 x-y+7 z=28$

## Problem 4.

Find the distance from the origin to the line

$$
x=2 t+2, y=t-3, z=t-4
$$

Answer: $\sqrt{\frac{55}{2}}$

## Problem 5.

Reduce the equation to one of the standard forms, classify the surface:

$$
x^{2}+2 y^{2}+3 z^{2}-2 x-4 y-6 z-15=0
$$

Answer: $\frac{(x-1)^{2}}{a^{2}}+\frac{(y-1)^{2}}{b^{2}}+\frac{(z-1)^{2}}{c^{2}}=1$, where $a^{2}=21, b^{2}=21 / 2, c^{2}=7$. It is an ellipsoid.

