MULTIVARIABLE CALCULUS MATH 2D

Midterm Exam II (with answers)

Problem 1.

A particle starts at the origin with initial velocity $\bar{i}+\bar{j}-\bar{k}$. Its acceleration is $\bar{a}(t) = 6t\bar{i}+12t^2\bar{j}-6t\bar{k}$. Find its position function.

Answer: $\bar{r}(t) =< t^3 + t,\ t^4 + t,\ -t^3 - t >$

Problem 2.

Let C be the curve of intersection of the parabolic cylinder $x^2 = 2y$ and the surface $3z = xy$. Find the exact length of C from the origin to the point $(2, 2, \frac{4}{3})$.

Answer: $\frac{10}{3}$

Problem 3.

At what point does the curve $y = -\ln x,\ 0 < x < +\infty$, have maximal curvature?

Answer: $\left(\ln \sqrt{2},\ \frac{1}{\sqrt{2}}\right)$

Problem 4.

Find the equation of the tangent plane to the surface $z = 3x^2 - y^2 + 2x$ at the point $(1, 0, 5)$.

Answer: $z - 5 = 8(x - 1)$

Problem 5.

Suppose $z = f(x, y)$, where $x = g(s, t), y = h(s, t)$, $g(1, 2) = 3$, $g_s(1, 2) = -1$, $g_t(1, 2) = 4$, $h(1, 2) = 6$, $h_s(1, 2) = -5$, $h_t(1, 2) = 10$, $f_x(3, 6) = 9$, and $f_y(3, 6) = -2$. Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial s}$ when $s = 1$ and $t = 2$.

Answer: $\frac{\partial z}{\partial s} = 1,\ \frac{\partial z}{\partial t} = 16$