

MULTIVARIABLE CALCULUS MATH 2D

Midterm Exam II (with answers)

Problem 1.

A particle starts at the origin with initial velocity $\bar{i} - \bar{j} - \bar{k}$. Its acceleration is $\bar{a}(t) = 6t\bar{i} + 12t^2\bar{j} - 6t\bar{k}$. Find its position function.

Answer: $\bar{r}(t) = \langle t^3 + t, t^4 - t, -t^3 - t \rangle$

Problem 2.

Let C be the curve of intersection of the parabolic cylinder $x^2 = 2y$ and the surface $3z = xy$. Find the exact length of C from the origin to the point $(3, \frac{9}{2}, \frac{9}{2})$.

Answer: $\frac{15}{2}$

Problem 3.

At what point does the curve $y = \ln x$, $0 < x < +\infty$, have maximal curvature?

Answer: $(\ln \sqrt{2}, -\frac{1}{\sqrt{2}})$

Problem 4.

Find the equation of the tangent plane to the surface $z = 3x^2 - y^2 + 2x$ at the point $(-1, 1, 0)$.

Answer: $z = -4(x + 1) - 2(y - 1)$

Problem 5.

Suppose $z = f(x, y)$, where $x = g(s, t)$, $y = h(s, t)$, $g(1, 2) = 3$, $g_s(1, 2) = -1$, $g_t(1, 2) = 4$, $h(1, 2) = 6$, $h_s(1, 2) = -5$, $h_t(1, 2) = 10$, $f_x(3, 6) = 8$, and $f_y(3, 6) = 7$. Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ when $s = 1$ and $t = 2$.

Answer: $\frac{\partial z}{\partial s} = -43$, $\frac{\partial z}{\partial t} = 102$