Section 16, problems 1, 2 and these problems:

Problem 1.

Determine the number of zeros of the polynomial

$$z^{100} + 50z^{50} + 100z^2 + 1$$

in the annulus $\{1 < |z| < 2\}$.

Problem 2.

Let *U* be a bounded open connected set, $\{f_n\}$ a sequence of continuous functions on the closure of *U*, analytic in *U*. Assume that $\{f_n\}$ converges uniformly on ∂U . Prove that $\{f_n\}$ converges uniformly on *U*.

Problem 3.

Find an explicit conformal map of the open set

$$\{0 < \text{Im} \, z < 1\} \setminus \{z = it, t \in [0, 1/2]\}$$

to the unit disc.

Problem 4.

Find the integral

$$\int_0^\pi \frac{d\theta}{3+2\cos\theta}.$$

Problem 5.

Let \mathcal{F} be the family of all analytic functions

$$f(z) = z + a_2 z^2 + a_3 z^3 + \dots$$

on the open unit disc, such that $|a_n| \leq n$ for each n. Prove that \mathcal{F} is a normal family.