## Complex Analysis, HW \# 6

Section 16, problems 1, 2 and these problems:

## Problem 1.

Determine the number of zeros of the polynomial

$$
z^{100}+50 z^{50}+100 z^{2}+1
$$

in the annulus $\{1<|z|<2\}$.

## Problem 2.

Let $U$ be a bounded open connected set, $\left\{f_{n}\right\}$ a sequence of continuous functions on the closure of $U$, analytic in $U$. Assume that $\left\{f_{n}\right\}$ converges uniformly on $\partial U$. Prove that $\left\{f_{n}\right\}$ converges uniformly on $U$.

## Problem 3.

Find an explicit conformal map of the open set

$$
\{0<\operatorname{Im} z<1\} \backslash\{z=i t, t \in[0,1 / 2]\}
$$

to the unit disc.

## Problem 4.

Find the integral

$$
\int_{0}^{\pi} \frac{d \theta}{3+2 \cos \theta} .
$$

## Problem 5.

Let $\mathcal{F}$ be the family of all analytic functions

$$
f(z)=z+a_{2} z^{2}+a_{3} z^{3}+\ldots
$$

on the open unit disc, such that $\left|a_{n}\right| \leq n$ for each $n$. Prove that $\mathcal{F}$ is a normal family.

