COMPLEX ANALYSIS

Practice Qualifying Exam

Wednesday, May 29, 2013 — 2:00pm - 4:30pm, Rowland Hall 340N

This Exam is for training purposes only. It will not influence you Math 220C final grade, and cannot substitute the actual Qualifying Exam in Complex Analysis in any way.

Problem	1	2	3	4	5	6	7	8	Σ
Points									

Student's name:

Problem 1.

Find a conformal transformation which maps G onto the unit disk $\mathbb{D}$, where

$$G = \left\{ z \in \mathbb{C} : |z| > 1 \text{ and } \left| z - \frac{1+i}{2} \right| < \frac{1}{\sqrt{2}} \right\}.$$

Problem 2.

Find the integral

$$\int_{|z|=\frac{3}{2}} \frac{4z^3+3z^2}{z^4+z^3+1}dz$$

Problem 3.

Find explicitly the group of conformal automorphisms of the domain

$$U = \{z \in \mathbb{C} \mid |z-1| > 1\}\,.$$

Problem 4.

If f is an entire function satisfying the estimate

$$|f(z)| \leq 1 + |z|^{2013+\sqrt{2}\sin|z|}, \quad z \in \mathbb{C},$$

show that f is a polynomial and determine the best upper bound for the degree of f.

Problem 5.

Evaluate the following integral:

$$\int_{-\infty}^{+\infty} \frac{dx}{x^4 + 4x^2 + 3}$$

Problem 6.

Let $f(z) = \sum_{n=0}^{\infty} a_n z^n$ be an entire function. Prove that for any $r \geq 0$ one has

$$\frac{1}{2\pi}\int_0^{2\pi} |f(re^{i\theta})|^2 d\theta = \sum_{n=0}^{\infty} r^{2n}|a_n|^2$$

Problem 7.

For which values of the parameter $\mu \in \mathbb{R},\ \mu \geq 0$, the following product converges uniformly on compact sets in $\mathbb{C}$?

$$\prod_{n=1}^{\infty} \left(\frac{1}{n^{\mu}} + \exp\left(\frac{1}{\sqrt{n}} \sin \frac{z}{n^{\mu}} \right) \right)$$

Problem 8.

TRUE or FALSE: Suppose $v : \mathbb{C} \to \mathbb{R}$ is a continuous function such that for any point $z \in \mathbb{C}$ and any $n \in \mathbb{N}$ we have $\frac{1}{2\pi} \int_0^{2\pi} v(z + \frac{e^{i\theta}}{n}) d\theta = v(z)$. Then v is harmonic.