Complex Analysis Math 220B

Final Exam (Sample)

Problem 1.

Let $U \subseteq \mathbb{C}$ be a convex open set, and $f : U \to \mathbb{C}$ be a holomorphic function. Prove that if Re $f'(z) > 0$ in U then f is a conformal mapping of U to $f(U)$.

Problem 2.

How many roots of the equation $z^4 + z^3 - 4z + 1 = 0$ are in the ring $1 < |z| < 4$?

Problem 3.

Let u and v be harmonic in $\mathbb{C}$ and assume that v is harmonic conjugate of u. Assume that

$$u^3 - 3uv^2 \geq 0$$

in $\mathbb{C}$. Prove that u and v are constants.

Problem 4.

Is there a function f holomorphic in the unit disc $D(0,1)$ and such that $|f(z)| = e^{|z|}$ there?

Problem 5.

TRUE or FALSE: If u is a harmonic function on an open set U and $p > 0$ then $|u|^p$ is subharmonic. Prove or give a counterexample.