Complex Analysis Math 220A

Midterm Sample Exam

Problem 1.

Find the largest disk centered at 1 in which the Taylor series for

$$\frac{1}{1+z^2} = \sum a_k (z-1)^k$$

will converge.

Problem 2.

Evaluate

$$\int_0^{2\pi} e^{e^{i\theta}} d\theta$$

Problem 3.

Prove that there is no entire analytic function such that

$$\bigcup_{n=0}^{\infty} \left\{ z \in \mathbb{C} \ : \ f^{(n)}(z) = 0 \right\} = \mathbb{R}.$$

Problem 4.

TRUE or FALSE? There is no entire function f such that for some real constants $C_1, C_2 > 0$ and all $z \in \mathbb{C}$

$$C_1(|z|^{\frac{7}{5}} + 1) \leq |f(z)| \leq C_2(|z|^{\frac{3}{2}} + 1).$$

Prove or give a counterexample.

Problem 5.

Find the radius of convergence R_1 of the series

$$\sum_{n=1}^{+\infty} \frac{z^n}{n^2}$$

and show the series converges uniformly on $\overline{D(0, R_1)}$. What is the radius of convergence R_2 of the derivative of this series? Does it converge uniformly on $\overline{D(0, R_2)}$?