

# COMPLEX ANALYSIS MATH 220A

## Final Exam (sample)

### Problem 1.

Evaluate the following integral:

$$\int_{\partial D(0,r)} \frac{dz}{(z-b)(z-a)^m}, \quad |a| < r < |b|, \quad m \text{ is arbitrary integer.}$$

### Problem 2.

Let  $f(z)$  be analytic on  $\mathbb{C} \setminus \{1\}$  and have a simple pole at  $z = 1$  with residue  $\lambda$ . Prove that for every  $R > 0$ ,

$$\lim_{n \rightarrow \infty} R^n \left| (-1)^n \frac{f^{(n)}(2)}{n!} - \lambda \right| = 0.$$

### Problem 3.

Evaluate the integral

$$\int_{-\infty}^{\infty} \frac{\cos kx}{1+x+x^2} dx$$

where  $k$  is a non-negative real number.

### Problem 4.

Suppose that  $f(z)$  and  $g(z)$  are non-constant entire functions such that  $(f \circ g)(z)$  is a polynomial. Show that both  $f(z)$  and  $g(z)$  are polynomials.

### Problem 5.

Let

$$f(z) = \sum_{n=0}^{\infty} a_n z^n = 1 + z + 2z^2 + 3z^3 + 5z^4 + 8z^5 + 13z^6 + 21z^7 + \dots$$

where the coefficients  $a_n$  are the Fibonacci numbers, i.e.  $a_0 = a_1 = 1$ ,  $a_n = a_{n-1} + a_{n-2}$  for  $n \geq 2$ . Show that  $f(z) = \frac{1}{1-z-z^2}$  for  $z \in D(0, \frac{\sqrt{5}-1}{2})$ .