COMPLEX ANALYSIS MATH 220A

Midterm Exam

Friday, November 7, 2014 — 9:00 am - 9:50 am

Problem	1	2	3	4	5	Σ
Points						

Student's name:

Problem 1.

Find explicitly

$$\left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)^{2014}$$

Problem 2.

Find the radius of convergence for the series:

$$\sum_{n=1}^{+\infty} \frac{z^{2n}}{n!} \quad \text{and} \quad \sum_{n=1}^{+\infty} \frac{z^{n!}}{2n}$$

Problem 3.

Is there an entire function f such that

$$f\left(\frac{1}{n}\right) = f\left(-\frac{1}{n}\right) = \frac{1}{n^3}$$

for all $n \in \mathbb{N}$? Justify your answer.

Problem 4.

Show that for any $R > 0$, there is N_R such that when $n > N_R$, the function

$$P_n(z) = 1 + z + \frac{z^2}{2!} + \ldots + \frac{z^n}{n!} \neq 0 \quad \text{for all} \quad |z| \leq R.$$

Problem 5.

Find all entire functions $f(z)$ on $\mathbb{C}$ satisfying

$$|f(z)| \leq |z|e^x, \quad z = x + iy \in \mathbb{C}.$$