COMPLEX ANALYSIS MATH 220A

Final Exam (sample)

Problem 1.

Evaluate the following integral:

$$\int_{\partial D(0,r)} \frac{dz}{(z-b)(z-a)^m}, \quad |a| < r < |b|, \quad m \text{ is arbitrary integer.}$$

Problem 2.

Let $f(z)$ be analytic on $\mathbb{C}\setminus\{1\}$ and have a simple pole at $z = 1$ with residue λ. Prove that for every $R > 0$,

$$\lim_{n\to\infty} R^n \left| (-1)^n \frac{f^{(n)}(2)}{n!} - \lambda \right| = 0.$$

Problem 3.

Evaluate the integral

$$\int_0^\infty \frac{1}{(1+x^2)x^{1/2}} dx$$

Problem 4.

Show that there is a holomorphic function defined in the set

$$\Omega = \{z \in \mathbb{C} \mid |z| > 4\}$$

whose derivative is

$$\frac{z}{(z-1)(z-2)(z-3)}.$$

Is there a holomorphic function on Ω whose derivative is

$$\frac{z^2}{(z-1)(z-2)(z-3)}?$$

Problem 5.

Let f be an entire function. Suppose that for each complex number a, the power series expansion

$$f(z) = \sum_{n=0}^{\infty} c_n (z-a)^n, \quad z \in \mathbb{C},$$

has at least one coefficient $c_n = 0$. Show that f is a polynomial.