COMPLEX ANALYSIS MATH 220A

Midterm Exam

Monday, October 29, 2012 — 2:00 pm - 2:50 pm

Problem	1	2	3	4	5	Σ
Points						

Student's name:

Problem 1.

Prove that for any $z_1, z_2 \in \mathbb{C}$

$$||z_1| - |z_2|| \leq |z_1 - z_2| \leq |z_1| + |z_2|. \tag{1}$$

When do equalities hold in (1)?

Problem 2.

Solve the equation (where $n \in \mathbb{N}$)

$$(1 - z)^n = z^n$$

Problem 3.

Let $p(z) = \sum_{l=0}^{n} p_l z^l$ be a polynomial bounded by 1 in modulus in the closed unit disc. Show that all $|p_l| \leq 1$ for $l = 0, \dots, n$.

Problem 4.

Compute, for $t \in \mathbb{R}$,

$$\lim_{n\to\infty} \frac{1 + e^{it} + e^{i2t} + \ldots + e^{int}}{n}$$

Problem 5.

Show that the series

$$\sum_{n=1}^{\infty} \left(\frac{1}{z-n} + \frac{1}{n} \right)$$

converges for every $z \notin \mathbb{N}$. Show that the convergence is uniform on any compact set which does not intersect $\mathbb{N}$.