Ergodic Theory, Math 211A, Midterm

Problem 1.

Prove that the decimal expansion of the number 2^n may begin with any finite combination of digits.

Problem 2.

Prove that if a measure-preserving transformation $T : (X, \mu) \to (X, \mu)$ has a generating partition ξ with k elements then $h_\mu(T) \le \log k$.

Problem 3.

Prove that the measure preserving transformations $\sigma_2 : (\Sigma_2, \mu_{1/2,1/2}) \to (\Sigma_2, \mu_{1/2,1/2})$ and $\sigma_3 : (\Sigma_3, \mu_{1/3,1/3,1/3}) \to (\Sigma_3, \mu_{1/3,1/3,1/3})$ are not metrically isomorphic.

Problem 4.

Let $\sigma_A : \Sigma_A \to \Sigma_A$ be a topological Markov chain with the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$. Find $\lim_{n\to\infty} \frac{1}{n} \log \left(\#Per_n(\sigma_A)\right)$.

Problem 5.

Consider the map $f : \mathbb{T}^2 \to \mathbb{T}^2$, $T(x, y) = (x + \alpha, x + y) (\mathrm{mod}\ 1)$, where $\alpha \notin \mathbb{Q}$. Formulate as many questions about this map (e.g. "Is Lebesgue measure the only invariant measure of f, i.e. is f uniquely ergodic?") and answer them.