DYNAMICAL SYSTEMS (FALL 2010)

Final Exam

Problem 1.

Consider the map $f : S^1 \to S^1$,

$$f(x) = \begin{cases} 4x \text{ (mod 1)}, & \text{if } x \in [0, \frac{1}{2}); \\ 6x \text{ (mod 1)}, & \text{if } x \in [\frac{1}{2}, 1). \end{cases}$$

Calculate $h_{\text{top}}(f)$.

Problem 2.

Consider the map $F_{\alpha,\beta} : \Sigma^2 \times S^1 \to \Sigma^2 \times S^1$, $\omega \in \Sigma^2$, $\varphi \in S^1$,

$$F_{\alpha,\beta}(\omega, \varphi) = \begin{cases} (\sigma(\omega), R_\alpha(\varphi)), & \text{if } \omega_0 = 0; \\ (\sigma(\omega), R_\beta(\varphi)), & \text{if } \omega_0 = 1. \end{cases}$$

For which pairs (α, β) the map $F_{\alpha,\beta}$ is transitive?

Problem 3.

Consider expending maps

$$E_2 : S^1 \to S^1,\ E_2(x) = 2x(\text{mod}1), \quad \text{and}$$

$$E_3 : S^1 \to S^1,\ E_3(x) = 3x(\text{mod}1).$$

Denote by F the product map, $F : \mathbb{T}^2 \to \mathbb{T}^2$, $F = E_2 \times E_3$. Is it possible to find a point $x \in \mathbb{T}$ such that $\omega(x)$ is homeomorphic to a circle? To a Cantor set? To a product of a circle and a Cantor set?

Problem 4.

Let Λ_i be a hyperbolic set of $f_i : U_i \to M_i$, $i = 1, 2$. Prove that $\Lambda_1 \times \Lambda_2$ is a hyperbolic set of $f_1 \times f_2 : U_1 \times U_2 \to M_1 \times M_2$.

Problem 5.

Prove that any contracting C^1-diffeomorphism of $\mathbb{R}$ is topologically conjugated to a linear contraction.