Real Analysis

Sample Final

Problem 1.

TRUE or FALSE: For any $n \times n$ matrix A which is close enough to Id_n there exists an $n \times n$ matrix X such that $A = X^2 + 3X + \mathrm{Id}_n$.

Problem 2.

Among all rectangular parallelepipeds with diagonal equal to 3 find the one with largest value of $x + 2y + 3z$, where x, y, z are the sides of parallelepiped.

Problem 3.

Find the area of the set $\{|y| \leq 1, \;\; |x - 10y| \leq \sqrt{1 - y^2}\}$.

Problem 4.

Let $y = (y_1(x), y_2(x))$ be a function of x defined by a system of equations

$$\begin{cases} y_1^2 + y_2^2 = x \\ y_1 = \sin 2\pi x + \cos 2\pi y_2, \end{cases}$$

where (x, y_1, y_2) is in a neighborhood of the point $(2, 1, 1)$. Compute $\frac{dy_1}{dx}$ and $\frac{dy_2}{dx}$ at the point $(2, 1, 1)$. Justify the existence of the derivative.

Problem 5.

A C^1-function $f : \mathbb{R}^2 \to \mathbb{R}^2$ has the derivative $DF(p) = \begin{pmatrix} a(p) & b(p) \\ c(p) & d(p) \end{pmatrix}$ with the following properties: $|a(p)| \leq 1, |b(p)| \leq 2|a(p)|, |c(p)| \leq |b(p)|, |d(p)| = |a(p)|$ for all $p \in \mathbb{R}^2$. Is it possible that $f(0) = 0$ and $f(1) = 2012$?