Real Analysis
Math 205C, Spring 2012

Practice Exam

Friday, May 25, 2012 — 1:00pm-3:30pm

Problem	1	2	3	4	5	6	7	8	Σ
Points									

Student's name:

Problem 1.

TRUE or FALSE? For any bounded collection of real numbers $\{a_{mn}\}_{m,n\in\mathbb{N}}$ we have

$$\limsup_{n\to\infty}(\limsup_{m\to\infty} a_{mn}) = \limsup_{m\to\infty}(\limsup_{n\to\infty} a_{mn})$$

Problem 2.

Which number is larger, π^3 or 3^π?

Problem 3.

Suppose $f : \mathbb{R} \to \mathbb{R}$ is continuous for $x \in \mathbb{R}$, differentiable for $x \in (-\infty, 0) \cup (0, \infty)$, and $\lim_{x \to 0} f'(x) = 2$. Show that f is differentiable at $x = 0$ and $f'(0) = 2$.

Problem 4.

Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function of two variables and assume that the restriction of f to any line in $\mathbb{R}^2$ is differentiable (i.e. for any $a, b, c, d \in \mathbb{R}$ the function of one variable $g(t) = f(at + b, ct + d)$ is differentiable). Is f continuous on $\mathbb{R}^2$? Prove or give a counterexample.

Problem 5.

Suppose (x_0, y_0, u_0, v_0) is a solution to the system of equations

$$\begin{cases} e^x \cos y + 2u - v = 0 \\ e^x \sin y - u + 2v = 0 \end{cases}$$

(a) Show that in a neighborhood of (x_0, y_0, u_0, v_0) the system can be solved for (x, y) as a function of (u, v);

(b) Show that in a neighborhood of (x_0, y_0, u_0, v_0) the system can be solved for (u, v) as a function of (x, y).

Problem 6.

Suppose $\{x_n\}$ is a sequence in a complete metric space (M, d) such that

$$\sum_{n \in \mathbb{N}} d(x_n, x_{n+1})$$

is a convergent series. Show that the sequence $\{x_n\}$ is convergent in M.

Problem 7.

Let $\{f_n\}$ be a sequence of real-valued continuous functions on $[0, 1]$ which converges pointwise on the interval $[0, 1]$. Suppose that f_n is continuously differentiable on $(0, 1)$ and that

$$\int_0^1 |f_n'(x)|^2 dx \leq 1, \quad n = 1, 2, 3, \ldots$$

Prove that

(a) $\{f_n\}_{n\in\mathbb{N}}$ is equicontinuous on $[0, 1]$;

(b) $\{f_n\}_{n\in\mathbb{N}}$ converges uniformly on $[0, 1]$.

Problem 8.

Find the maximal area of all triangles that can be inscribed in an ellipse with semiaxes a and b.