# REAL ANALYSIS
# MATH 205C, SPRING 2012

**HW# 7**

## Problem 1.

Use the Divergence Theorem to calculate the flux of the vector field $v(x, y, z) = (x^3, y^3, z^3)$ through the unit sphere.

## Problem 2.

Calculate the flux of the vector field $v(x, y, z) = (x, y, z)$ through the outwardly oriented surface obtained by removing the cube $[1, 2]\times[1, 2]\times[1, 2]$ from the cube $[0, 2]\times[0, 2]\times[0, 2]$.

In the problems 3.-5. find a vector field $v$ in $\mathbb{R}^3$ whose divergence is given by function $f$:

## Problem 3.

$f(x, y, z) = 1$

## Problem 4.

$f(x, y, z) = x^2y$

## Problem 5.

$f(x, y, z) = \sqrt{x^2 + z^2}$

## Problem 6.

Let $S$ be the surface obtained by rotating the curve

$$\begin{cases} x = \cos u, \\ z = \sin 2u, \end{cases} \qquad -\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$$

around the $z$-axis. Find the flux of the vector field $v(x, y, z) = (0, 0, z)$ through $S$.

## Problem 7.

Find the volume of the region inside $S$, where $S$ is the surface from Problem 6.

## Problem 8.

Evaluate the flux of the vector field $v(x, y, z) = (x + y, z^2, x^2)$ through the hemisphere $\{x^2 + y^2 + z^2 = 1,\ z \geq 0\}$ (note that the surface is not closed!).

## Problem 9.

Consider a surface $S = \{x^2 + y^2 + z^2 - 2xyz = 1,\ |x| \leq 1, |y| \leq 1, |z| \leq 1\}$. Prove that the flux of the vector field $v(x, y, z) = (x, y, z)$ through $S$ is equal to the flux of the vector field $w(x, y, z) = (x - \sin y \sin z, \cos y + \sin z, (2 + \sin y)z)$ through $S$.

## Problem 10.

Let $S_C$ be the compact connected component of the set $\{x^2 + y^2 + z^2 - 2xyz = C\}$, $C \in (0, 1)$. Suppose that $C^*$ is such that the volume of the region inside of $S_{C^*}$ is equal to $1/10$. Evaluate the integral

$$\int_{S_{C^*}} \frac{x^2 + y^2 + z^2 - 3xyz}{\sqrt{(x - yz)^2 + (y - zx)^2 + (z - xy)^2}} dA$$