Real Analysis
Math 205C, Spring 2012

HW# 6

Problem 1.

Calculate $\int_\gamma \omega$, where $\omega = xdx + ydy + zdz$ and $\gamma : [0,1] \to \mathbb{R}^3$, $\gamma(t) = (\cos 2\pi t, \sin 2\pi t, t)$.

Problem 2.

Calculate $\int_\gamma \omega$, where $\omega = \frac{1}{\sqrt{2}}dx + \frac{1}{\sqrt{2}}dy$ and $\gamma : [0,1] \to \mathbb{R}^2$, $\gamma(t) = (t,t)$. Compare the result with the length of γ.

Problem 3.

Is it possible to find a 1-form ω in $\mathbb{R}^2$ such that $\int_\gamma \omega$ is equal to the length of γ for any smooth curve γ?

Problem 4.

Consider the following 2-cell in $\mathbb{R}^3$:

$$\varphi : [0,1] \times [0,1] \to \mathbb{R}^3,\ \varphi(u,v) = (\sin \pi u \cos 2\pi v, \sin \pi u \sin 2\pi v, \cos \pi u).$$

Show that the range of φ is the unit sphere in $\mathbb{R}^3$, and check that $\partial\varphi = 0$.

Problem 5.

Calculate $\int_\varphi \omega$, where

$$\omega = \frac{xdy \wedge dz + ydz \wedge dx + zdx \wedge dy}{(x^2+y^2+z^2)^{3/2}}$$

and

$$\varphi : [0,1] \times [0,1] \to \mathbb{R}^3,\ \varphi(u,v) = (\sin \pi u \cos 2\pi v, \sin \pi u \sin 2\pi v, \cos \pi u).$$

Problem 6.

Consider the form $\omega = \sin 2x \sin y dx + \sin^2 x \cos y dy$ in $\mathbb{R}^2$. Is it closed? Exact?

Problem 7.

Consider the form $\omega = \frac{xdy - ydx}{x^2+y^2}$ in $\mathbb{R}^2 \setminus \{(0,0)\}$. Is it closed? Exact?

Problem 8.

Consider the form $\omega = yzdx + xzdy + xydz$ in $\mathbb{R}^3$. Is it closed? Exact?

Problem 9.

Consider the form $\omega = \frac{xdy\wedge dz + ydz\wedge dx + zdx\wedge dy}{(x^2+y^2+z^2)^{3/2}}$ in $U \subset \mathbb{R}^3, U = \{(x,y,z) \mid 1 < x^2+y^2+z^2 < 9\}$. Is it closed? Exact?

Problem 10.

The following problem was given for the final exam:"Find $\int_C \omega$, where $\omega = dx \wedge dz$ and $C = \{(x,y,z) \mid x^2+y^2 = 1, 0 \leq z \leq 1, y \geq 0\}$" One student gave the answer "2", another one gave the answer "-2". Who has made a mistake?