Real Analysis
Math 205C, Spring 2012

HW# 5

Problem 1.

Find the volume of the set

$$\{(x,y,z)\mid -1\le z\le 1,\ 4(x-\sin z)^2+(y-\cos z)^2\le 1\}$$

Problem 2.

Find the area of the set

$$\{(x,y)\mid -1\le x\le 1,\ |y-\sqrt{1-x^2}|\le |x|\}$$

Problem 3.

Find $\int_{\mathbb{R}^2} f(x,y)dxdy$, where

$$f(x,y)=\begin{cases}\cos\sqrt{x^2+y^2}, & \text{if } x^2+y^2\le\frac{\pi^2}{4};\\ 0, & \text{if } x^2+y^2>\frac{\pi^2}{4}.\end{cases}$$

Problem 4.

Let $f(x,y,z)=z(x^2+y^2)$. Find $\int_K f$, where $K=\{0\le z\le 1,\ x^2+y^2\le 1\}$.

Problem 5.

Find $\int_C \omega$, where $\omega = \sin y\cos x dx+\sin x\cos y dy$, and C is a unit circle (oriented counterclockwise).

Problem 6.

Find $\int_C \omega$, where $\omega = xdx+xy^2dy$, and C is an interval connecting $(0,0)$ and $(2,2)$.

Problem 7.

Suppose α and β are smooth 1-forms in $\mathbb{R}^2$ such that for any path C we have $\int_C \alpha = \int_C \beta$. Prove that $\alpha = \beta$ (i.e. coefficients of α and β are equal).

Problem 8.

Assume that ω is a 1-form in $\mathbb{R}^2$ such that $\int_C \omega = 0$ for any closed curve C. Prove that ω is exact.

Problem 9.

Let D be a unit disc in $\mathbb{R}^2$ centered about the origin. Calculate explicitly $\int_D d\omega$ and $\int_{\partial D} \omega$ if $\omega = xdy$.

Problem 10.

TRUE OR FALSE: If ω is a k-form and k is odd then $\omega \wedge \omega = 0$. What if k is even?