Real Analysis
Math 205C, Spring 2012

HW# 4

In problems 1-4 determine the maxima and minima of f on the surface (or curve).

Problem 1.

$f(x, y, z) = x + y + 2z$ on the surface $x^2 + y^2 + z^2 = 3$.

Problem 2.

$f(x, y, z) = xy$ on the curve $3x^2 + y^2 = 6$.

Problem 3.

$f(x, y, z) = x^2 - y^2$ on the surface $x^2 + 2y^2 + 3z^2 = 1$.

Problem 4.

$f(x, y, z) = 8x - 4z$ on the surface $x^2 + 10y^2 + z^2 = 5$.

Problem 5.

Find minimum of $\sum_{i=1}^{5} x_i^2$ subject to constraints $\begin{cases} x_1 + 2x_2 + x_3 = 1 \\ x_3 - 2x_4 + x_5 = 6 \end{cases}$

Problem 6.

Find the extreme values of $f(x, y) = 2x^2 + 3y^2 - 4x - 5$ on the region $x^2 + y^2 \leq 16$.

Problem 7.

What is the smallest possible value of the sum of squares of elements of a matrix from $SL(2, \mathbb{R})$? Find all the matrices where this minimum is attained.

Problem 8.

Among all rectangles with diagonal 1 find the one with the largest difference between its area and the square of its smaller side.

Problem 9.

Is it true that any compact set $K \subset \mathbb{R}^2$ is Riemann measurable (i.e. its boundary is a set of zero measure)?

Problem 10.

Is it true that any bounded open set $U \subset \mathbb{R}^2$ is Riemann measurable (i.e. its boundary is a set of zero measure)?