Real Analysis

Sample Final

Problem 1.

Show that $\int_0^\infty x^{-\alpha} \sin x dx$ exists for $0 < \alpha < 2$.

Problem 2.

a) For what real values of x does the series

$$f(x) = \sum_{k=1}^{\infty} \frac{x^k}{k^2}$$

converge?

b) Compute $f'(1/3)$. Justify your answer.

Problem 3.

Suppose f is differentiable on $[a, b]$, $f(a) = 0$, and there is $M > 0$ such that $|f'(x)| \leq M|f(x)|$ for each $x \in [a, b]$. Prove $f \equiv 0$ on $[a, b]$.

Problem 4.

Suppose $f_1 \in R[0, M]$, and $f_{n+1}(x) = \int_0^x f_n(t)dt$, $n \in \mathbb{N}$. Prove that $f_n \rightrightarrows 0$ on $[0, M]$.

Problem 5.

Let F be a collection of all finite linear combinations of function of the form $f_a(x) = e^{ax}$, $a \in \mathbb{R}$, restricted to $[0, 1]$. Check that F is an algebra of functions. Is it dense in $C[0, 1]$?