Real Analysis
Math 205B, Winter 2012

Midterm Exam

Tuesday, February 21, 2012 — 10:00am-10:50am

Problem	1	2	3	4	5	Σ
Points						

Student's name:

Problem 1.

For which values of the real parameters α, β the function

$$f_{\alpha,\beta}(x) = \begin{cases} 0, & x = 0; \\ x^\alpha \sin x^\beta, & x > 0 \end{cases}$$

has bounded variation?

Problem 2.

Suppose that $\{f_\alpha\}_{\alpha\in A}$, $f_\alpha : [0,1] \to \mathbb{R}$, is an equicontinuous family. Suppose also that $f_\alpha(0) = 0$ for all $\alpha \in A$. Prove that $\{F_\alpha\}_{\alpha\in A}$, $F_\alpha : [0,1] \to \mathbb{R}$, $F_\alpha(x) = \int_0^x f_\alpha(t)dt$, is also an equicontinuous family.

Problem 3.

For $x \geq 1$ denote by $l(x) \in \mathbb{N}$ the number of digits in the natural number $[x]$. Prove that the improper integral

$$\int_1^\infty \frac{dx}{x\,(l(x))^2}$$

converges.

Problem 4.

Suppose $f : \mathbb{R} \to \mathbb{R}$ is continuous, $f_n(x) = f(nx)$, $n \in \mathbb{N}$, and the family of functions $\{f_n\}_{n\in\mathbb{N}}$ is equicontinuous on $[-1, 1]$. What conclusion can you draw about f?

Problem 5.

Prove that the space $C^1[0,1]$ (the space of continuously differentiable functions on $[0,1]$ with the metric $\|f-g\|_{C^1} = \max(\|f-g\|_\infty, \|f'-g'\|_\infty)$ is connected.