REAL ANALYSIS

HW# 8

Remark: *This is the last homework this quarter, so it contains only 5 problems to allow sufficient time to prepare for the final exam.*

Chapter 15, problems 8, 9, and also the following problems:

Problem 1.

Suppose the trigonometric polynomial

$$T(x) = \frac{a_0}{2} + \sum_{k=1}^{n} (a_k \cos kx + b_k \sin kx)$$

is represented as

$$T(x) = \sum_{m=-n}^{n} c_m e^{imx}, \quad c_m \in \mathbb{C}.$$

Find the relation between a_k, b_k, and c_k.

Problem 2.

Under what condition on the coefficients $\{c_m\}$ the trigonometric polynomial

$$T(x) = \sum_{m=-n}^{n} c_m e^{imx}, \quad c_m \in \mathbb{C}, \quad x \in \mathbb{R},$$

is real-valued?

Problem 3.

Let $f(x)$ be Riemann integrable on $[0, 2\pi]$, and let

$$g(t) = \int_0^{2\pi} f(x) \sin tx \, dx, \quad t \in \mathbb{R}.$$

a) Prove that $g(t)$ is uniformly continuous on $\mathbb{R}$;

b) Prove that $\lim_{n \to \infty} g(n) = 0$.