REAL ANALYSIS

HW# 7

Chapter 14, problem 56; Chapter 15, problems 1, 2, 3, 5, 6, 7,

and also the following problems:

Problem 1.

For a given linear functional $L : C[0,1] \to \mathbb{R}$ find explicitly a function $\alpha \in BV[0,1]$ such that $L(f) = \int_0^1 f d\alpha$.

a) $L(f) = f(0) + f(1)$

b) $L(f) = \int_{1/3}^{2/3} f(x)dx$

c) $L(f) = \sum_{n=1}^{\infty} \frac{1}{2^n} f\left(\frac{1}{n}\right)$

d) $L(f) = \sum_{n=1}^{\infty} (-1)^n \int_{\frac{1}{n+1}}^{\frac{1}{n}} f(x)dx$

Problem 2.

Find the Fourier series for $f \in C[-\pi, \pi]$, $f(x) = |x|$.

Problem 3.

Find the Fourier series for

$$f(x) = \begin{cases} 0, & \text{for } -\pi \leq x < 0; \\ 1, & \text{for } 0 \leq x \leq \pi. \end{cases}$$