REAL ANALYSIS

HW# 5

Chapter 14, problems 6, 7, 12, 18, 25,

and also the following problems:

Problem 1.

Does the integral $\int_0^\infty \sin(x^2)dx$ converge?

Problem 2.

Suppose $f : [0, +\infty) \to \mathbb{R}$ is non-negative and continuous, and $\int_0^\infty f(x)dx$ converges. Does it imply that $f(x) \to 0$ as $x \to \infty$?

Problem 3.

Does the integral $\int_0^\infty |\sin(x^2)|dx$ converge?

Problem 4.

Suppose $f : [0, +\infty) \to \mathbb{R}$ is continuous, and there exists a limit

$$\lim_{x\to\infty} \left(f(x) + \int_0^x f(t)dt \right) = L < +\infty.$$

Prove that $f(x) \to 0$ as $x \to \infty$.

Problem 5.

Consider the "dyadic ruler function" $g(x) : [0, 1] \to \mathbb{R}$,

$$g(x) = \begin{cases} \frac{1}{2^n}, & \text{if } x = \frac{k}{2^n}, \text{ where } k \text{ is an odd natural number;} \\ 0, & \text{otherwise.} \end{cases}$$

What is its set of discontinuities? At which points its oscillation $\geq \mu > 0$? Is it Riemann integrable?