Real Analysis
Math 205A, Fall 2011

Midterm Sample

Problem 1.

Define a sequence $(a_1, a_2, ...a_n, ...)$ recursively by setting $a_1 = 1, a_2 = 3$, and $a_{n+2} = (a_{n+1} + 2a_n)/3$ for $n \geq 1$. Prove that the sequence $\{a_n\}$ converges, and compute its limit.

Problem 2.

Prove that $\mathbb{R}^2$ is equivalent (i.e. has the same cardinality) to $\mathbb{R}^1$.

Problem 3.

Prove that any subset of a metric space can be written as
a) a union of closed sets;
b) an intersection of open sets.

Problem 4.

Which of the following metric spaces are separable (explain!):
a) l_1; b) l_2; c) l_∞; d) $\mathbb{R}^n$.

Problem 5.

Let $f : [a, b] \to \mathbb{R}$ be a real-valued function defined on a closed bounded interval $[a, b]$ in $\mathbb{R}$. We define the graph of the function f to be set Γ_f of points $(x, y) \in \mathbb{R}^2$ such that $x \in [a, b]$ and $y = f(x)$.
Prove or Disprove: The function f is continuous on $[a, b]$ if, and only if, the set Γ_f is a closed subset of $\mathbb{R}^2$.