Real Analysis
Math 205A, Fall 2011

Homework 2, due October 14, 2011 in class

Problem 1.

Let $\{a_n\}$ be a sequence of real numbers such that $\lim_{n\to\infty} a_n = a$. Prove that

$$\lim_{n\to\infty} \frac{a_1 + a_2 + \ldots + a_n}{n} = a.$$

Problem 2.

Calculate $\lim_{n\to\infty}(\sqrt{n^2+n} - n)$.

Problem 3.

If $s_1 = \sqrt{2}$, and $s_{n+1} = \sqrt{2+\sqrt{s_n}}$, $n \in \mathbb{N}$, prove that $\{s_n\}$ converges, and $s_n < 2$ for all $n \in \mathbb{N}$.

Problem 4.

If $\sum a_n$ converges, and if $\{b_n\}$ is monotonic and bounded, prove that $\sum a_n b_n$ converges.

Problem 5.

Prove that the product of two absolutely converging series converges absolutely.

Problem 6.

Test for convergence or divergence:

$$\sum_{n=1}^{\infty} \frac{\log n}{n^{\frac{3}{2}}}$$

Problem 7.

For which values of $p > 0$ does the series

$$\sum_{n=1}^{\infty} \frac{n^p}{n!}$$

converges?

Problem 8.

Find the values of $x \in \mathbb{R}$ for which the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x-1)^n}{2^n n^2}$$

converges.

Problem 9.

Prove that the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n (n-1)}{n^2+1}$$

converges. Does it converge absolutely?

Problem 10.

If $\sum_{n=1}^{\infty} a_n$ is a converging series of positive terms, show that $\sum_{n=1}^{\infty} a_n^p$ also converges for every $p > 1$.