MATH 118A, FALL 2010

Final exam (sample), v.1

Problem 1.

Find all singular points of the vector field

$$\begin{cases} \dot{x} = \sin y \\ \dot{y} = \sin x \end{cases}$$

and determine their stability (both Lyapunov and asymptotic).

Problem 2.

Plot the phase portrait for the Newton equation

$$\ddot{x} = -4x^3 + 4x$$

Problem 3.

Let $f : [0,1] \to [0,1]$ be a continuous mapping. Is it possible that f has a periodic orbit of prime period 2011, but does not have a periodic orbit of prime period 2010?

Problem 4.

Find all fixed points of the map $f : \mathbb{R}^1 \to \mathbb{R}^1$, $f(x) = 2x - x^2$, and determine their stability.

Problem 5.

Find the box counting dimension of the subset of $[0,1]$ consisting of real numbers without digit "7" in their decimal expansion.

Final exam (sample), v.2

Problem 1.

Show that for every value of the real parameter μ the singular point $(0,0)$ of the system

$$\begin{cases} \dot{x} = 2x + (1+\mu)y \\ \dot{y} = (1-\mu)x + y \end{cases}$$

is not Lyapunov stable.

Problem 2.

Plot the phase portrait of the system

$$\begin{cases} \dot{x} = y \\ \dot{y} = x^2 - 1 \end{cases}$$

Problem 3.

Show that the first order system $\dot{x} = \mu - x^3 + x$ undergoes two saddle-node bifurcations as μ varies, and find the values of μ at the bifurcation points.

Problem 4.

Let $f : \mathbb{R}^1 \to \mathbb{R}^1$ be a map given by

$$f(x) = \begin{cases} 4x, & \text{if } x \leq \frac{1}{2}; \\ -4x+4, & \text{if } x > \frac{1}{2}. \end{cases}$$

Describe the set

$$C = \{x \in \mathbb{R} \mid \{f^n(x)\}_{n\in\mathbb{N}} \text{ is bounded } \},$$

and find its box counting dimension.

Problem 5.

Find all fixed points of the map $f : \mathbb{R}^1 \to \mathbb{R}^1$, $f(x) = x^2 - 2$, and determine their stability.