

# MATH 118A, FALL 2010

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## Sample Midterm

$$\begin{cases} \dot{x} = -2x^2 + 1 + y^2 \\ \dot{y} = x - y \end{cases} \quad (1)$$

### Problem 1.

Find all singular points of the vector field (1).

### Problem 2.

Find linearization at every singular point of (1).

### Problem 3.

Find eigenvalues of linearizations of singular points of (1), and determine stability and the type of the corresponding linear system.

### Problem 4.

For the Newton equation

$$\ddot{x} = x \sin x$$

draw the phase portrait of the corresponding non-linear system.

### Problem 5.

For which values of the parameter  $\mu$  the point  $(0, 0)$  is Lyapunov stable? Asymptotically stable?

$$\begin{cases} \dot{x} = x^2(x - \mu) \\ \dot{y} = y(y - 2\mu) \end{cases} \quad (2)$$