Lines in the Curriculum: A persistent obstruction to Achievement Connecting Algebra and Geometry

11 AM, March 29, 2010: Conference Room at MIND Institute

Summary

Today's Problem: : Use 9th grade algebra and 10th grade geometry to describe lines in 3-space. A student who understands lines, understands a lot.

1. **Part I**: Description of Lines in the K-14 Curriculum
2. - I.A: There are many lines in the curriculum
   - I.B: The Point of Lines: Direction and Orientation
   - I.C: What is a line?: A Persistent Curricular Problem
3. **Part II**: California Framework Awareness?
4. - II.A. Where the Framework stood at the end of the '90s
   - II.B. What are lines worth?
   - II.C. The Howard Thompson Story
   - II.D. Standards and Curricular Goals

Part I: Description of Lines in the K-14 Curriculum
I.A: There are many lines in the curriculum

Rubric: What is a line in —?
[In practice the material is usually taught earlier than given here.]

1. In Kindergarten? Answer: A "straight" cut by scissors in the hands of a teacher or coordinated child.
2. In 4th grade? Answer: The result of a steady hand in pulling a pencil across a ruler.
3. In 8th-9th grade? Answer: An expression like $y = 2x + 3$.
4. In 10th grade? Answer: It is undefined, except by properties: Two points determine a unique line.
5. In 11th-12th grade? Answer: An expression like $y = mx + b$.

And it both continues and gets used in other courses

1. In High School Chemistry? Example: A boundary between two phases of a substance in a plot of temperature versus pressue.
2. In 1st year Calculus/physics? Answer: $y = mx + b$: $m = \frac{d}{dx}(f(x))|_{x=x_0}$ for $f(x)$ at a point x_0. Approximating 4th grade line to the graph of f near x_0.
3. In 2nd year Calculus or in physics? Answer:

$$\{(x_0, y_0, z_0) + t(u, v, w) \mid t \in \mathbb{R}\}.$$

A *parametric line*: t is a *parameter*.

I.B: The Point of Lines: Direction and Orientation

Suppose I want to guide you to somewhere where you are walking across a field. I might use time and direction.

- Walk in the (u_1, v_1, w_1) direction for t_1 minutes at the rate $\sqrt{U_1^2 + v_1^2 + w_1^2} = r_1$ feet/minute.
- Then, walk in the (u_2, v_2, w_2)direction for t_2 minutes at the rate $\sqrt{U_1^2 + v_1^2 + w_1^2} = r_2$ feet/minute.
- How would I explain this to you? Answer: I might use my left arm to point. Maybe you are on an airplane (or a proton) and I'm directing you to a target up in the air.
- What picture might I have?: Answer: At time $t = 0$ you are at (x_0, y_0, z_0). You end up in one minute at $(x_0 + u, y_0 + v, z_0 + z)$, in 2 minutes at $(x_0 + 2u, y_0 + 2v, z_0 + 2z)$, 3 minutes, etc.

If $y = 2x + 3$ is a line in (x, y)-space,
then is $z = 2x + 3y + 4$ a line in (x, y, z)-space?

1. The first class I asked this had 57 students.
2. - All 57 students answered "Yes!"
   - What was their reasoning?: Answer: It looks like the formula for a line, and it has the right number of variables.
3. I then asked if there was a reason it might not be a line.
4. - From Euclid: If two points are in a plane, the line *determined by them* has all its points (lies) on the plane.
   - 2nd tack: Find points on $S = \{(x, y, z) \mid z = 2x + 3y + 4\}$. What is the meaning of taking $x = 0$, then taking $y = 0$?

Using the principle: Two points determine a line.

1. Setting $x = 0$ confines points *on* S to a coordinate plane.

$$S_{x=0} = \{(0, y, z) \mid z = 3y + 4\}.$$

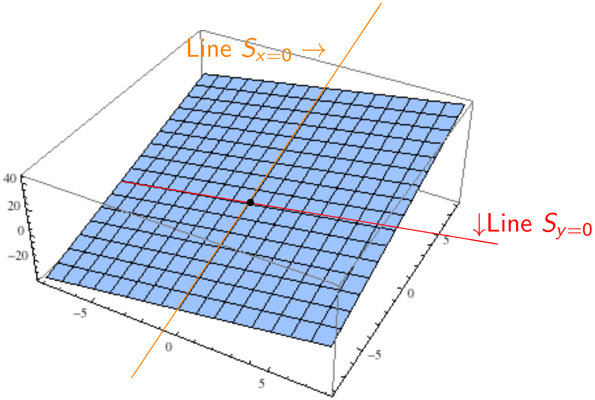
   Same with $y = 0$. Do the points so confined look like lines?
2. In each coordinate plane there is a traditional 9th grade line. Two distinct (not parallel) lines on S meet at a point on S.
3. Does x, y and z appearing to first power give S one property of a plane? Example: The points $(1, 1, 9)$ and $(-1, -1, -1)$ are on S. Then, all points on the line

$$L((1, 1, 9) : (-1, -1, -1))$$

   they determine are also on S. How would you check? Hint: How would you write expressions for those points?
4. Euclid had no definition for a *plane*: No joke!
5. How students perform: With similar questions, students bring out slopes. Such training won't support 3-D thinking.

Mathematica produces S

Two lines on a Plane



In 218 page Framework, here are all references to lines or planes:

- p. 83: "In this strand [Geometry] students . . . investigate 2-dimensional and 3-dimensional space by exploring shape, area and volume; lines, angles, points and surfaces."
- p. 120: In listing of the NCTM standards for grades 5-8, for Standard 9, Algebra: "Develop confidence in solving linear equations, using concrete, informal and formal methods." There was no mention of lines in Standard 12, Geometry.
- p. 123: Unifying Ideas for Middle Grades: "Central concept is ... proportion. ... proportionality, slope, linear functions, ... the ration of *rise* over *run* is the slope of the line."
- p. 126: ... *maps* in middle school involved plane *geometry*. "in their work with maps, students explore . . . *paths* and how to specify straight paths by . . . [using] . . . distance."

Assumption in High School: Lines were mastered in Junior High

- pp. 134–174 gives the contents of the High School Curriculum. Almost no mention of lines.
- p. 143: "Students need to see the analogy between familiar frames of reference used to locate places (street patterns, building interiors . . .) and . . . coordinate geometry."
- Yet, it was tessellations, packing problems and fractals that were brought up as relevant topics.

Pretty, fancy froth.

- The text didn't mention lines, and the difficulties in representing them in 3-space.

II.B. What are lines worth?

Strongest statement relating HS courses, pps 155–157: 3 headings
Connections between functions and algebra
Connections between functions and geometry
Connections between algebra and geometry
Lines were never mentioned as a persistent difficulty.

- 2nd Year Calculus [seems to stand] all alone.
- Result: Almost *all* students fail: Fail to see how mathematics and engineering, physics, chemistry, economics, social sciences – that use line as a tool – relate.
- Another day's Lesson: Parametric lines give meaning to solving equations. Even if you don't have a method to solve particular equations, there still is a well-defined meaning to solving them.

II.C. The Howard Thompson Story

http://www.math.uci.edu/ mfried/edlist-tech/gold02-08-98.html

- Administrators balked at my insisting sophomore courses impeded progress for so many students. The larger cohort of freshman calculus seemed like more "bang for the buck."
- Until I brought up Howard Thompson, they didn't realize I was talking about a total bottleneck for minority students ever participating in Mathematics, Science or Engineering.
- Sloan Foundation Grant toward an acknowledged problem: *Nearly* 100% minority student wipeout from 1st quarter sophomore Calculus.
- My statistic: 51 "black" [our word then] students took the 1st quarter vector calc. in a 10 year period. At first I said," none," but then changed that, offhandedly, to "one" got through.

An enlightened administrator

- This was a public presentation among system-wide UC administers, arranged by Dennis Galligani. He asked from the back, "Well, who was the student who got through?"
- My answer, "Howard Thompson." Dennis repeated that, "Howard Thompson? He was the only black student who got through the first quarter of vector calculus?" Every administrator in the room knew Howard Thompson.
- http://www.math.uci.edu/ mfried/edlist-tech/gold02-08-98.pdf Interactive E-Mail Assessment, MAA Vol. on Assessment, B. Gold, S.Z. Keith, and W.A. Marion, eds., Assessment in Undergraduate Mathematics, MAA Notes #49, Wash. DC, 1999, 80–84.

II.D. Standards and Curricular Goals

- "No Child Left Behind" measures the number of students who are proficient at each grade level. It requires states to adopt "challenging academic standards" in reading and math – see Title 1 – but leaves it to states to define "challenging."
- Analogy: Let teachers repeat the text book in class, but leave developing *analyzing skills* to students on their own.
- Result: States set their standards at widely varied levels. Obama would measure each student's academic growth, regardless of their starting performance level.

Action Suggestions: Obama versus Many blue-ribbon committees

- Obama/Duncan: Recommend Congress overhaul NCLB. Require states to adopt "college- and career-ready standards" in reading and mathematics.
- Curricular areas must identify persistent difficulties.
- What Blue-Ribbons Want: Start from scratch, get better teachers, do first years perfectly. Then, everything will be fine.
- Analogy: The Greeks were brilliant and so everything marched forward from that time.
- Action: Common-standards effort has produced a draft.
- Assessment is everything: That requires replacing NCLB's much-criticized school rating system, known as *adequate yearly progress*, with a new accountability system.