

Midterm Examination

Print your name: _____

Print your ID #: _____

You have 50 minutes to solve the problems. Good luck!

1. **A.** For $\varepsilon > 0$ compute the solution $y_\varepsilon(t)$ of

$$y'' - (2 + \varepsilon)y' + (1 + \varepsilon)y = 0, \quad y(0) = 0, \quad y'(0) = 1,$$

and determine the limit as $\varepsilon \rightarrow 0$.

- B.** For $\varepsilon > 0$ compute the solution $y_\varepsilon(t)$ of

$$y'' - 2y' + (1 + \varepsilon^2)y = 0, \quad y(0) = 0, \quad y'(0) = 1,$$

and determine the limit as $\varepsilon \rightarrow 0$.

2. A. Solve

$$\begin{cases} y'''' - 2y'' + 2y' + y = e^{2t}, \\ y(0) = 1, y'(0) = 0, y''(0) = 0, y'''(0) = 0. \end{cases}$$

B. Solve

$$\begin{cases} y'''' - 2y'' + y = \sin(2t), \\ y(0) = 1, y'(0) = 0, y''(0) = 0, y'''(0) = 0. \end{cases}$$

3. Classify every point of the following equations into ordinary, regular singular or irregular singular point. Justify your answer.

A. (i) $(x - 1)^2 \tanh(3x)y'' + (x^2 - 1)y' + (x - 1)^2y = 0,$

(ii) $(x - 1)^2(x + 3)y'' + (x - 4)y' + (x + 2)y = 0,$

(iii) $y'' + \frac{1}{(x+1)^2}y' + \frac{1}{x+1}y = 0.$

B. (i) $(x + 2)^2 \tanh(x)y'' + (x + 2)^2y' + (x^4 - 4)^2y = 0,$

(ii) $(x - 1)(x + 4)^2y'' + (x + 3)y' + (x - 2)y = 0,$

(iii) $y'' + \frac{1}{(x-1)}y' + \frac{1}{(x-1)^2}y = 0.$

4. Find the recurrence relation for the coefficients of the series solution about $t = 0$ and the first four terms in the expansion of two linearly independent solutions of

A. $(1 + t)y'' - y = 0$

B. $(1 - t)y'' + y = 0$

What is the radius of convergence of the series at least?

5. Find the exponents at the singularity for

A. $(e^x - e^{-x})y'' + y' + y = 0$

B. $(e^x - 1)y'' + \frac{1}{2}y' + y = 0$

How does the singular solution behave at $x = 0$?