Final Examination

Print your name: ________________ ________________

Print your ID #: ________________________________

You have 2 hours to solve the problems. Good luck!

1. Solve the equation and determine how long its solution exists:

A. $\dot{x} = x^3\,,\ x(0) = 1\,.$

B. $\dot{x} = -x^{-3}\,,\ x(0) = -1\,.$

2. Solve the equation:

A. $x'' + 9x' + x = e^{-3t}\,,\ x(0) = 0\,,\ x'(0) = 1\,.$

B. $x'' - 9x' + x = e^{3t}\,,\ x(0) = 1\,,\ x'(0) = 0\,.$

3. Indicate which of the following equations are linear (l) and which are nonlinear (n) by circling your answer:

A.	$x'' + e^t x' - x = \cos(t)$	l n
	$x'' + \cos(x')t - 3x = e^t$	l n
	$x''' + 5x = \sin(x)$	l n
	$x' = e^t \tanh(x)$	l n
	$x''' + 1/x = e^t \tanh(t)$	l n
B.	$x'' + e^x t - x = \sin(t)$	l n
	$x'' - \sin(t)x + 5x = \cosh(t)$	l n
	$x'''' + x'' - x = \sin(t)$	l n
	$x' = e^x \tanh(t)$	l n
	$x''' - 2/x = \cos^2(t)$	l n

4. Find the general solution of:

A. $x'''' + 2x''' - 2x' - x = 0\,.$

B. $x'''' - 2x''' + 2x' - x = 0\,.$

5. Solve the equation:

A. $x''' + t^2 x = 0\,,\ x(0) = 1\,,\ x'(0) = 0\,,\ x''(0) = 0\,.$

B. $x''' + tx = 0\,,\ x(0) = 0\,,\ x'(0) = 1\,,\ x''(0) = 0\,.$

6. For the following equations classify $t = 0$ into ordinary, regular singular or irregular singular point. Justify your answer and determine the exponents at the singularity for any regular singular point.

A. (i) $tx'' + t^2 x' + t^3 x = 0\,,$

(ii) $t^2 x'' + t^2 x' + tx = 0\,,$

(iii) $t^3 x'' + x = 0\,.$

B. (i) $t^3 x'' + t^2 x' + tx = 0\,,$

(ii) $t^3 x'' + x' + t^3 x = 0\,,$

(iii) $(1+t)^2 x'' + x/e^t = 0\,.$

7. Solve the equation:

A. $x''' + x = -h_1(t)\,,\ x(0) = 0\,,\ x'(0) = 0\,,\ x''(0) = 1\,.$

B. $x''' - x = h_1(t)\,,\ x(0) = 0\,,\ x'(0) = 0\,,\ x''(0) = 1\,.$

Recall that $h_1(t) := \begin{cases} 0\,, & t < 1 \\ 1\,, & t \geq 1 \end{cases}\,.$

8. Solve the system:

A. $x' = \begin{bmatrix} 3/2 & 0 & 1/2 \\ 0 & 5 & 0 \\ 1/2 & 0 & 3/2 \end{bmatrix} x + \begin{bmatrix} 0 \\ t \\ 0 \end{bmatrix}\,,\ x(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

B. $x' = \begin{bmatrix} 3/2 & 0 & 1/2 \\ 0 & 5 & 0 \\ 1/2 & 0 & 3/2 \end{bmatrix} x\,,\ x(0) = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$