Mid Term Examination

You have 50 minutes to solve 5 problems of your choice out of the 6 problems formulated below. You are not allowed to use a calculator. Please read the questions carefully before starting answering them. Good luck!

1. Find the general solution to the following equations:
$$\text{(a) } y'' + 2y' + y = t\,e^{-t}\,,\ t \in \mathbb{R}$$
$$\text{(b) } y'' - ty' = t\,,\ t \in \mathbb{R}$$

2. Find the solution y of the following logistic equation
$$y' = (\alpha - \beta y)\,y\,,\ t > 0\,,\ y(0) = y_0 > 0$$
for $\alpha,\,\beta > 0$. Compute $\lim_{t \to \infty} y(t)$ and discuss the solution's convexity in terms of the initial value y_0.

3. You have an amount K_0 USD that you would like to invest. You are offered a fund which yields 8% annually. The management fees amount to 5 USD/month.

(a) How long would it take for your initial investment to double?

(b) Assume you have 2000 USD that you would like to invest for 10 years. Another fund offers an annual return of 10% and costs of 120 USD/year. Which fund would you choose to maximize your profit?

[*Use continuous compounding as in class and $e^{0.8} \approx 2.2$ as well as $e \approx 2.7$.*]

4. Solve
$$y' = 1 - y^2\,,\ y(0) = y_0\,,$$
and compute $\lim_{t \to \infty} y(t)$ in terms of y_0.

5. The evolution of a fish population is described by
$$y' = y - F\bigl(1 + \cos(t)\bigr)\,,\ t > 0$$
where F represents the allowed fishing quota. What is the maximal quota which still prevents an initial population of 1000 fish from going extinct?

6. For $\alpha > 0$ and a continuous function g find the solution of
$$y'' - \alpha y = g(t)\,,\ t \in (0,1)\,,\ y(0) = y(1) = 0\,.$$