Midterm Examination

Print your name: _____ _____

Print your ID #: _____

You have 50 minutes to solve the problems. Good luck!

1. Draw a contour map of the given function.

   A. $f(x,y) = \sqrt{x^2/4 + y^2}\, e^{-x^2/4 - y^2}$, $(x,y) \in \mathbb{R}^2$.

   B. $f(x,y) = \frac{x^2 + y^2/4}{1 + x^2 + y^2/4}$, $(x,y) \in \mathbb{R}^2$.

2. Compute the given limits if they exist.

   A. $\lim_{(x,y)\to(0,0)} \frac{x^2(1+y) - y^2(x-1)}{x^2 + y^2}$

   B. $\lim_{(x,y)\to(0,0)} \frac{x^4(y-1) + y^4(x-1)}{x^4 + y^4}$

3. Compute $\partial_{xx} f$, $\partial_{yy} f$ and $\partial_{zz} f$ for the given function f.

   A. $f(x,y,z) = \sqrt{1 + x^2 + y^2 + z^2}$, $(x,y,z) \in \mathbb{R}^3$.

   B. $f(x,y,z) = \log\left(1 + x^2 + y^2 + z^2\right)$, $(x,y,z) \in \mathbb{R}^3$.

4. Find an equation for the tangent plane to the graph of f at the given point.

   A. $f(x,y) = e^{-x^2 - 2y}$, $(x_0, y_0, z_0) = (1, 0, 1/e)$.

   B. $f(x,y) = \sin\left(x^2 - 2y\right)$, $(x_0, y_0, z_0) = (\sqrt{\pi}, 0, 0)$.

5. Let $u(x,y)$ be a differentiable function. Compute $\partial_s + \partial_t$ of

   A. $u(2s + t, 2s - t)$.

   B. $u(2s + t, 2t - 2s)$.