

Assignment 10

1. Let T be a c_0 -semigroup on a Banach space E . Prove that there exist constants $M \geq 1$ and $\omega \in \mathbb{R}$ such that

$$\|T(t)\|_{\mathcal{L}(E)} \leq M e^{\omega t}.$$

2. For a c_0 -semigroup T on a Banach space E define

$$\xi(t, x) = T(t)x, \quad t \in [0, \infty), \quad x \in E$$

and prove that the following are equivalent:

- (i) $\xi(\cdot, x)$ is differentiable.
- (ii) $\xi(\cdot, x)$ is right differentiable.

3. Show that the translation semigroup T on $BUC(\mathbb{R})$ defined through

$$T(t)f(\cdot) = f(\cdot - t), \quad f \in BUC(\mathbb{R})$$

is strongly continuous and compute its generator.

4. Let $A \in \mathcal{G}(E)$, $x \in E$ and $f \in C^{1-}([0, \infty) \times E, E)$ and prove that

$$\begin{cases} \dot{u} + Au = f(t, u), & t > 0 \\ u(0) = x \end{cases}$$

has a unique local mild solution $u(\cdot, x) \in C([0, t^+(x)), E)$ for some $t^+(x) > 0$.

[Hint: Use Banach Fixed-Point Theorem combined with the Variation-of-Constants-Formula]

5. Let $A \in \mathbb{C}^{n \times n}$ and show that

$$e^{-tA} = \frac{1}{2\pi i} \int_{\partial \mathbb{B}(0, R)} e^{\lambda t} (\lambda + A)^{-1} d\lambda,$$

where $R > 0$ is such that $\sigma(-A) \subset \mathbb{B}(0, R)$ and the integration is counterclockwise.

[Hint: Show that both sides of the equation satisfy the same ODE and prove that they have the same initial value by reducing the problem to the case where A is a Jordan block]

The Homework is due Friday, February 5 2010