MATH 205 WINTER TERM 2006

Midterm Examination

Print your name: ________________ ________________

Print your ID #: ________________________________

You have 50 minutes to solve the problems. Good luck!

1. Let $\alpha \in \mathrm{B}([a,b])$ for $a < b \in \mathbb{R}$ and assume that

$$\int_a^b f(x)\, d\alpha(x) = 0$$

for any choice of monotone function $f \in \mathrm{B}([a,b])$. Show that α must be constant.

2. Let $f \in \mathcal{R}\big([a,b]\big)$ and compute

$$\lim_{n\to\infty} \frac{b-a}{n} \sum_{k=1}^{n} f\big(a + k\frac{b-a}{n}\big)$$

justifying why convergence actually takes place. Use the result to show that

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{n}{k^2+n^2} = \frac{\pi}{4}\,.$$

3. Let $0 \leq f \in \mathrm{C}([a,b],\mathbb{R})$ and show that

$$\lim_{n\to\infty}\Big(\int_a^b f(x)^n\,dx\Big)^{\frac{1}{n}} = \max_{x\in[a,b]} f(x)\,.$$

4. Consider the sequence of functions $(f_n)_{n\in\mathbb{N}}$ given by

$$f_n(x) := \sqrt{x^2 + \frac{1}{n^2}}\,,\ x \in \mathbb{R}\,.$$

What is the pointwise limit of the sequence? Is the convergence uniform? Justify your answers.

5. Let $(f_n)_{n\in\mathbb{N}}$ be a sequence of increasing real-valued functions defined on the interval $[a, b]$ which converges pointwise to $f \in \mathrm{C}([a, b])$. Show that f is increasing and that the convergence is uniform.