Assignment 24

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1. For which values of the constants do the following equations determine a C^1-surface?
   (a) $x^2 + y^2 - z^2 = c_1$, $x^2 + y^2 - z^2 = c_2$.
   (b) $xyz = c$.
   Compute tangent and normal spaces of these surfaces at each point where possible.

2. Let $M \subset \mathbb{R}^n$ be a m-dimensional C^1-manifold and $f \in C^1(M, \mathbb{R})$. Show that the function f possesses a smooth extension F to a neighborhood $U_x \in \mathcal{U}_{\mathbb{R}^n}(x)$ for every point $x \in M$.

3. Let $m \leq n$ and show that the set a $m \times n$-matrices of rank m is open in the space $\mathbb{R}^{m\times n}$.

4. Show that $\{M \in \mathbb{R}^{n\times n} \mid \det(M) = 1\}$ is a C^1-manifold of dimension $n^2 - 1$.

5. Find maxima and minima of $f(x, y, z) = x^2 + 4y^2 - z^2$ on the unit sphere $\mathbb{S}^2$.

Homework due by Thursday, June 1 2006