Assignment 20

1. Let $R = [a,b] \times [c,d]$ and $f \in \mathrm{C}(R,\mathbb{R})$ be given. Prove that the two dimensional Riemann integral of f as defined in class exists.

2. Let $\gamma \in \mathrm{C}^1([0,1],\mathbb{R}^n)$ and prove that $\gamma\big([0,1]\big)$ has content zero.

3. Prove that the graph of f defined through
$$f(x) = \sin(1/x)\,,\ x \in (0,1]\,,$$
has content zero.

4. Let B_f be the body of revolution obtained by rotating the graph of $f \in \mathrm{C}\big([-1,1],(0,\infty)\big)$ about the x-axis in $\mathbb{R}^3$. Compute its volume.

5. Let $\mathcal{GL}_n(\mathbb{R})$ denote the collection of invertible matrices of size $n \times n$ for $n > 1$. Assume that the function f is such that the integral
$$\int_{\mathcal{GL}_n(\mathbb{R})} f(x)|\det x|^{-n}\,dx$$
exists and prove that
$$\int_{\mathcal{GL}_n(\mathbb{R})} f(x)|\det x|^{-n}\,dx = \int_{\mathcal{GL}_n(\mathbb{R})} f(xy)|\det x|^{-n}\,dx\,,\ \forall\, y \in \mathcal{GL}_n(\mathbb{R})\,.$$

Homework due by Thursday, May 4 2006.