Assignment 17

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1. Prove continuity of the mapping
$$K : \mathrm{C}\big([0,1],\mathbb{R}^n\big) \to \mathrm{C}\big([0,1],\mathbb{R}^n\big)\,,\ f \mapsto K(f)$$
defined through
$$K(f)(x) := \int_0^x e^{f(y)}\,dy\,,\ x \in [0,1]\,.$$

2. Let (M, d_M) be a metric space. Show that
$$d_{x_0} : M \to \mathbb{R}\,,\ x \mapsto d_M(x, x_0)$$
is continuous.

3. Let (M, d_M) and (N, d_N) be metric spaces and assume that N is complete. Define $d_{\mathrm{B}(M,N)}$ through
$$d_{\mathrm{B}(M,N)}(f,g) = \sup_{x\in M} d_N\big(f(x), g(x)\big)\,,\ f,g \in \mathrm{B}(M,N)$$
and prove that
$$\big(\mathrm{B}(M,N), d_{\mathrm{B}(M,N)}\big)$$
is a complete metric space. Hereby we set
$$\mathrm{B}(M,N) := \{f : M \to N \mid f \text{ is bounded}\}\,.$$

4. Formulate and prove the fact that the composition $f \circ g$ of continuous functions f, g defined on metric spaces is continuous and give an example where the composition is continuous but nor f or g is continuous.

5. Find an example of a connected set which is not pathwise connected.

Homework due by Thursday, April 13 2006.