Practice Final

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1. [Least squares] Given points $(x_i, y_i) \in \mathbb{R}^2$, $i = 1, \ldots, n$, find the straight line $y = mx+b$ that best fits these points in that it minimizes
$$f(m, b) := \sum_{i=1}^{n} (mx_i + b - y_i)^2 .$$
Show that such a line always exists regardless of the given set of points.

2. Find the points on the ellipse $x^2 + 2y^2 = 1$ closest to and farthest from the origin.

3. Find an equation for the line tangent to the intersection curve of the surfaces $x^3 + 3x^2y^2 + y^2 + 4xy - z^2 = 0$ and $x^2 + y^2 + z^2 = 11$ at the point $(1, 1, 3)$.

4. Consider the function
$$f : \mathbb{R}^2 \to \mathbb{R}\,,\ (x, y) \mapsto x^3 + xy + y^3 + 1$$
and find the points $(x, y) \in \mathbb{R}^2$ for which the set $f^{-1}\big(f(x, y)\big)$ is a 1-dimensional manifold embedded in $\mathbb{R}^2$. Draw the set of points for which that is not the case.

5. Give a quadratic approximation to $f(x, y) = e^x \cos(y)$, $(x, y) \in \mathbb{R}^2$, at the origin.

Not due.