

## Assignment 6

---

1. Let  $I \subset \mathbb{R}$  be an interval and assume that  $f : I \rightarrow \mathbb{R}$  is monotone. Show that  $f$  is continuous if  $f(I)$  is also an interval.
2. Let  $f \in C(K, \mathbb{R})$  for some compact set  $K \subset \mathbb{R}$  and assume that  $f > 0$ . Show that  $1/f$  is uniformly continuous.
3. Let  $D_f, D_g \subset \mathbb{R}$ ,  $f \in C(D_f, \mathbb{R})$ ,  $g \in C(D_g, \mathbb{R})$  and assume that  $g(D_g) \subset D_f$ . Show that  $f \circ g \in C(D_g, \mathbb{R})$ .
4. Let  $x_0 \in D \overset{\circ}{\subset} \mathbb{R}$  and  $f \in C^1(D, \mathbb{R})$ . Prove that
$$a_{x_0} : D \rightarrow \mathbb{R}, x \mapsto f(x_0) + f'(x_0)(x - x_0)$$
is the best affine approximation to  $f$  as  $x \rightarrow x_0$ .

The Homework is due Monday, November 18