Assignment 5

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1. Let C_1 and C_2 be compact subsets of $\mathbb{R}$. Show that
$$C_1 + C_2 = \{x + y \mid x \in C_1 \text{ and } y \in C_2\}$$
is also compact.

2. Give examples of a continuous function which is not Hölder continuous of any exponent and of a Hölder continuous function which is not Lipschitz.

3. Let $f \in \mathrm{C}(\mathbb{R})$. Then
$$[f = \alpha] = \{x \in \mathbb{R} \mid f(x) = \alpha\} \text{ is closed } \forall \alpha \in \mathbb{R}$$
$$[f > \alpha] = \{x \in \mathbb{R} \mid f(x) > \alpha\} \text{ is open } \forall \alpha \in \mathbb{R}$$
Show that
$$\partial[f > \alpha] \subset [f = \alpha]\,.$$

4. Let $g, h \in \mathrm{C}(\mathbb{R})$ and prove that $f : \mathbb{R} \to \mathbb{R}$ given by
$$f(x) := \begin{cases} g(x)\,, & x \leq a \\ h(x)\,, & x > a \end{cases}$$
is continuous iff $g(a) = \lim_{x \to a+} h(x)$.

The Homework is due Tuesday, November 12